Pixtile® Art

Ars est celare artem -- Horace

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(and off-hours algorithmic artist)

Lockheed Martin Space Systems Company

We begin our story

A long time ago (15 years) in a laboratory far away (Palo Alto), a programmer made a mistake and got himself into…

Good Trouble!

This wild adventure began in 1993.

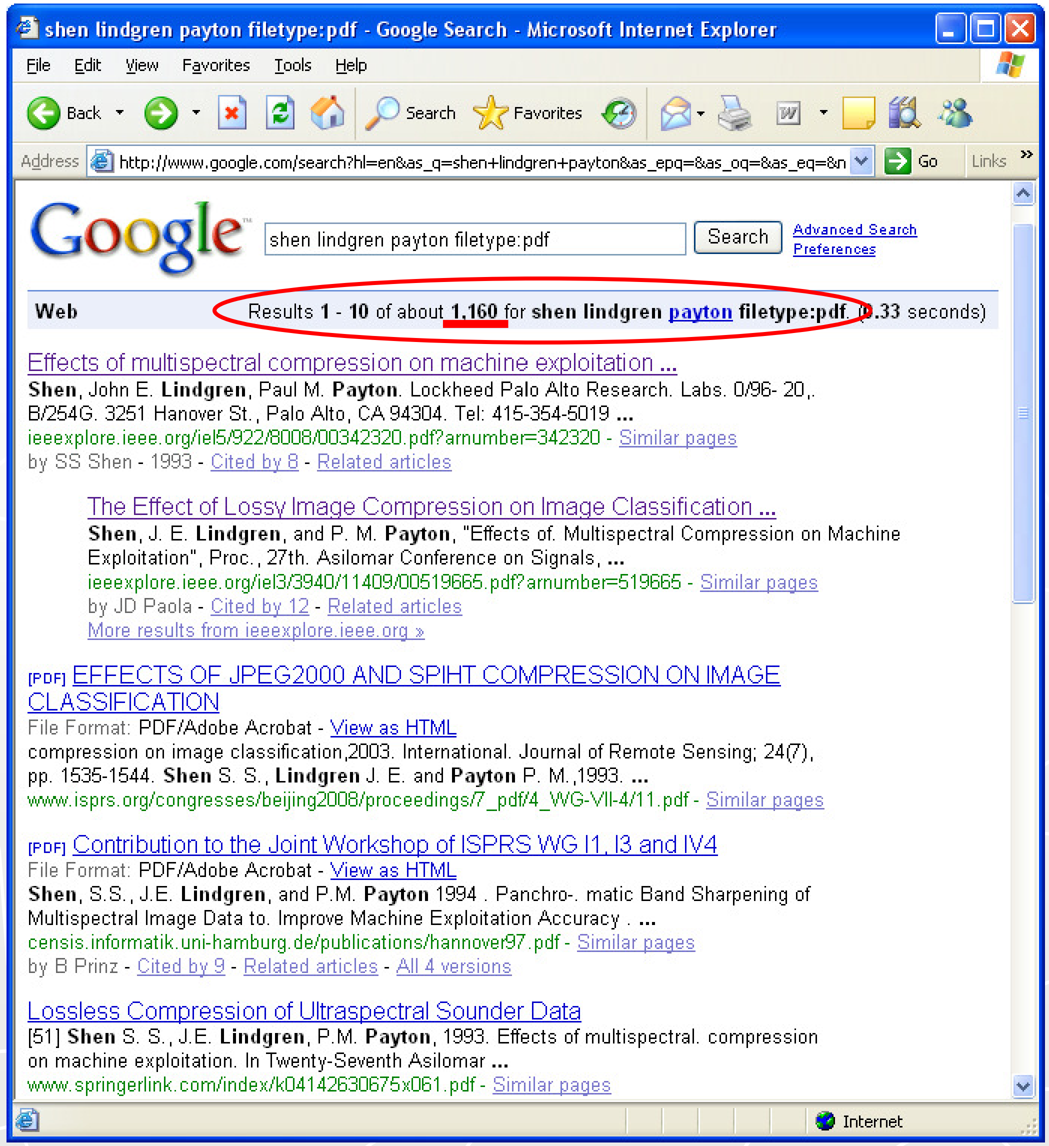
I worked on a Defense Landsat Program Office (DLPO) contract to study the impact of lossy compression (e.g., JPEG) on algorithms used to analyze multi-spectral satellite images.

How does degrading an image impact machine vision tasks like finding edges, identifying trees and turbid areas, and classifying areas into feature categories?

My job was quantifying the impact of image quality loss on texture, which is core to segmenting images into regions.

Back then, this was cutting-edge.

Ultimately, the result of our work became *a de facto standard.*



Our work is now seminal in the field.

I blame Haralick

Textural Features for Image Classification

ROBERT M. HARALICK, K. SHANMUGAM, AND ITS'HAK DINSTEIN

This is the 1973 paper defining what 'texture' is.

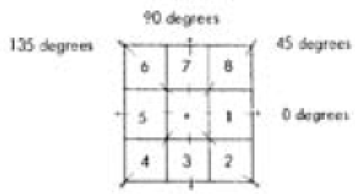


Fig. 1. Resolution cells 1 and 5 are 0° (horizontal) nearest neighbors to resolution cell *; resolution cells 2 and 6 are 135° nearest neighbors; resolution cells 3 and 7 are 90° nearest neighbors; and resolution cells 4 and 8 are 45° nearest neighbors to *. (Note this information is purely spatial, and has nothing to do with gray-tone values.)

It's defined in a 3x3 pixel neighborhood.

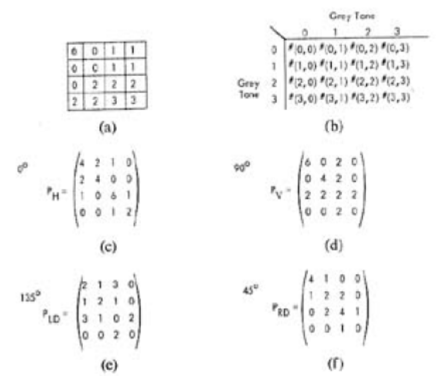


Fig. 3. (a) 4 × 4 image with four gray-tone values 0–3. (b) General form of any gray-tone spatial-dependence matrix for image with gray-tone values 0–3. $\#(i,j)$ stands for number of times gray tones i and j have been neighbors. (c)–(f) Calculation of all four distance 1 gray-tone spatial-dependence matrices.

Forming co-occurrence matrices is a key step.

$$P(i,j,d,0^\circ) = \#\{((k,l),(m,n)) \in (L_y \times L_x) \times (L_y \times L_x) \mid k-m=0, |l-n|=d, I(k,l)=i, I(m,n)=j\}$$

$$P(i,j,d,45^\circ) = \#\{((k,l),(m,n)) \in (L_y \times L_x) \times (L_y \times L_x) \mid (k-m=d, l-n=-d) \text{ or } (k-m=-d, l-n=d), I(k,l)=i, I(m,n)=j\}$$

$$P(i,j,d,90^\circ) = \#\{((k,l),(m,n)) \in (L_y \times L_x) \times (L_y \times L_x) \mid |k-m|=d, l-n=0, I(k,l)=i, I(m,n)=j\}$$

$$P(i,j,d,135^\circ) = \#\{((k,l),(m,n)) \in (L_y \times L_x) \times (L_y \times L_x) \mid (k-m=d, l-n=d) \text{ or } (k-m=-d, l-n=-d), I(k,l)=i, I(m,n)=j\} \quad (1)$$

where $\#$ denotes the number of elements in the set.

This stuff isn't easy!

Textural Features

1) *Angular Second Moment:*

$$f_1 = \sum_i \sum_j \{p(i,j)\}^2.$$

2) *Contrast:*

$$f_2 = \sum_{n=0}^{N_g-1} n^2 \left\{ \sum_{\substack{i=1 \\ |i-j|=n}}^{N_g} \sum_{j=1}^{N_g} p(i,j) \right\}.$$

3) *Correlation:*

$$f_3 = \frac{\sum_i \sum_j (ij)p(i,j) - \mu_x \mu_y}{\sigma_x \sigma_y}$$

where μ_x, μ_y, σ_x, and σ_y are the means and standard deviations of p_x and p_y.

More crunching is needed.

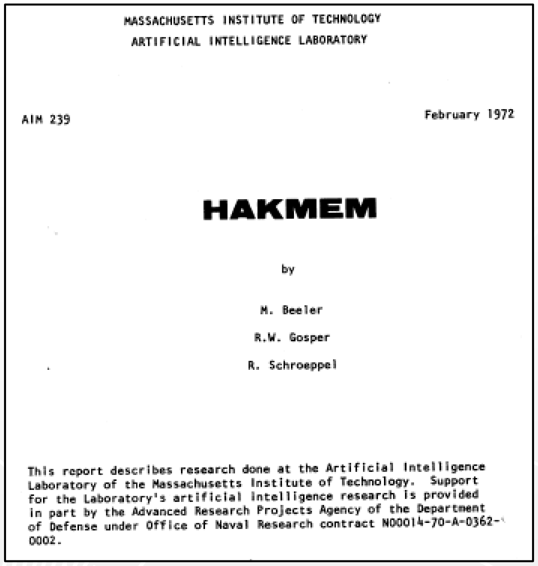
It's bad enough to read this…

…it's hard enough to code this…

…but how do you <u>test</u> this?

With an image having known statistical properties!

The exclusivity of XOR


MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

AIM 239 February 1972

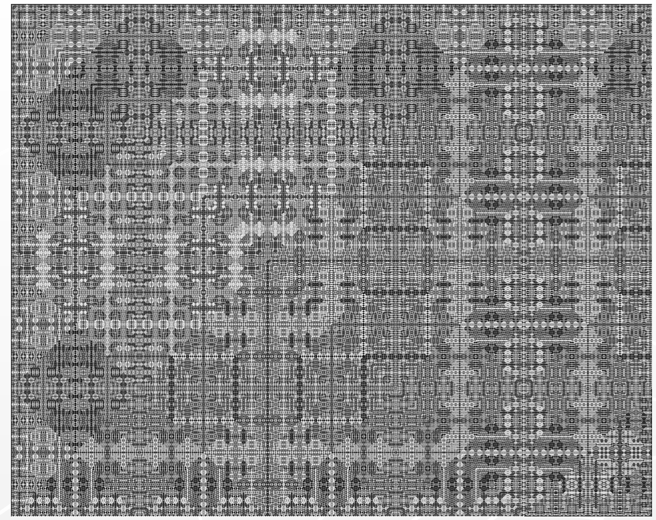
HAKMEM

by

M. Beeler
R.W. Gosper
R. Schroeppel

This report describes research done at the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology. Support for the Laboratory's artificial intelligence research is provided in part by the Advanced Research Projects Agency of the Department of Defense under Office of Naval Research contract N00014-70-A-0362-0002.

'HAKMEM' is a legendary report from MIT done in 1972 with invaluable 'hacker tricks' from a cadre of geniuses.



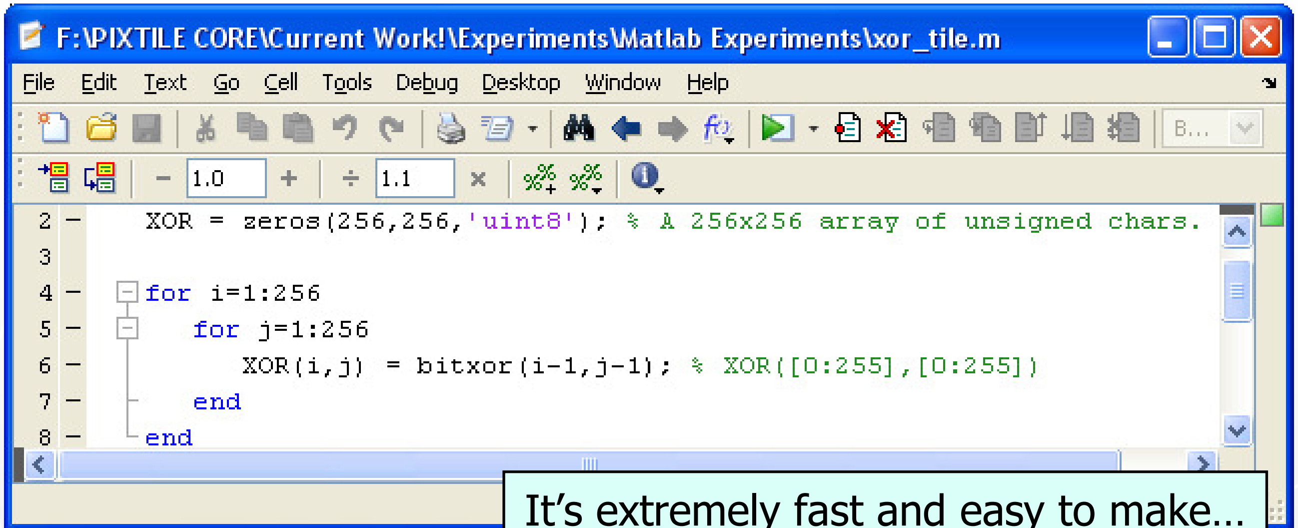
XOR (Boolean exclusive-OR) produces a highly regular pattern…a well-understood texture!

[This is the original rendering of Item #147.]

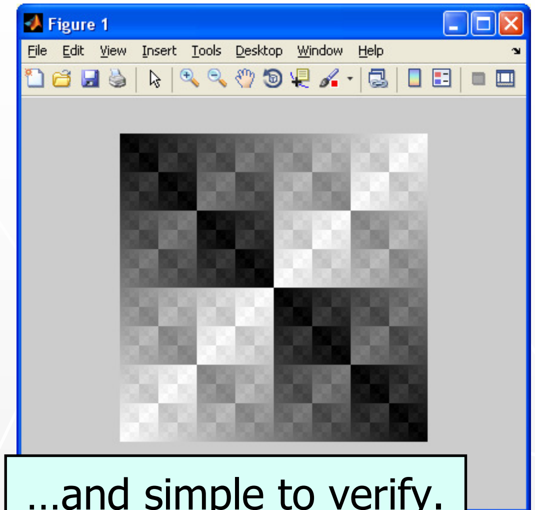
ITEM 147 (Schroeppel):
Munching squares is just views of the graph Y = X XOR T
for consecutive values of T = time.

Buried deep in HAKMEM is this little gem of wisdom.
It describes a perfect test image for texture!

The Perfect Test Image

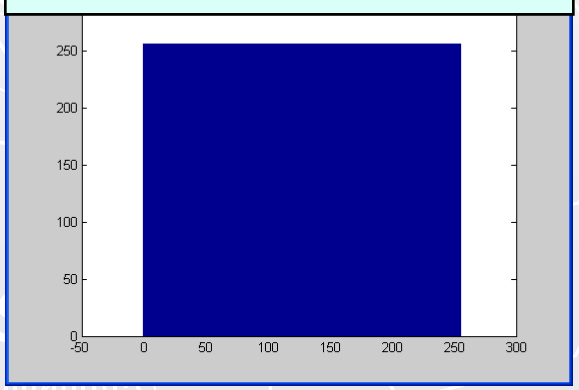


It's extremely fast and easy to make…

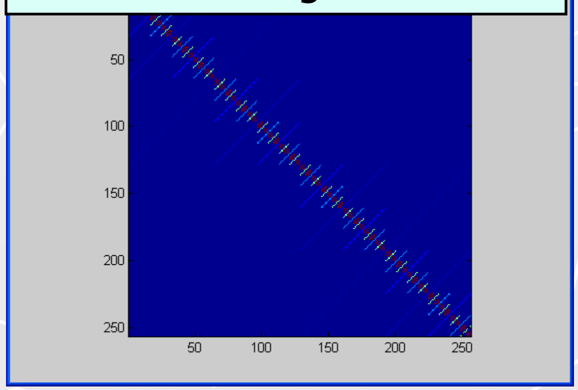


…and simple to verify.

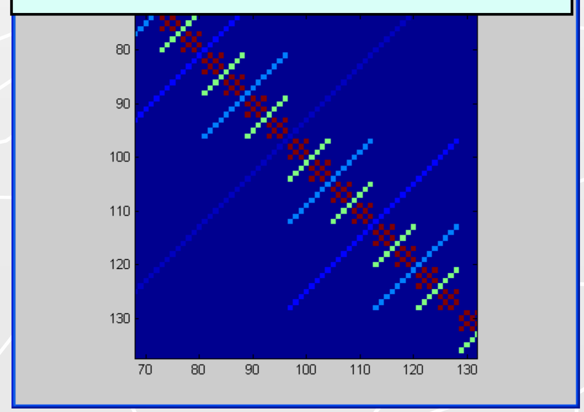
It has a uniform histogram that is indistinguishable from white noise…



…and a co-occurrence matrix with entries on the main diagonal…



…with the main diagonal entries all well-known.



I generated the XOR image using Metrowerks CodeWarrior on a Macintosh PowerBook 170*.

Then I examined it using NIH Image.

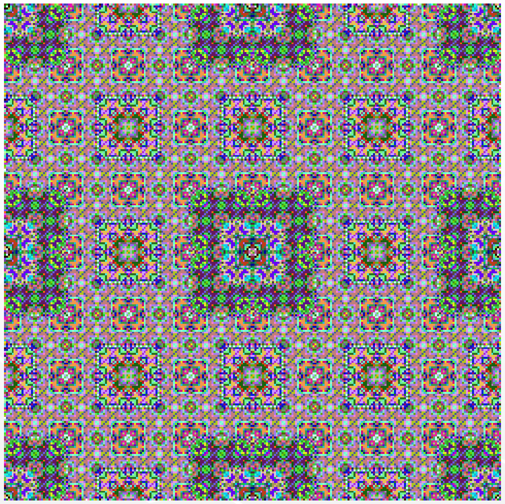
And then I hit the wrong button.

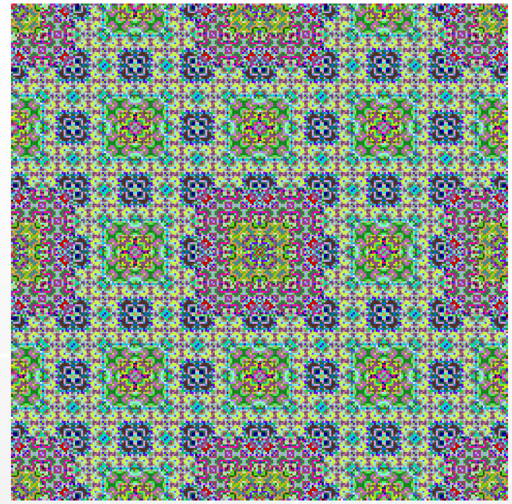
Instead of clicking on the 'histogram' button, I clicked on 'random pseudo-color' instead.

What I saw blew my mind.

***I'm an old Mac-head: Apple ///, PowerBook 170, PowerBook 180c, Apple G3, PowerBook G4 (and this was 1993)**

Like casting a net upon an infinite sea…



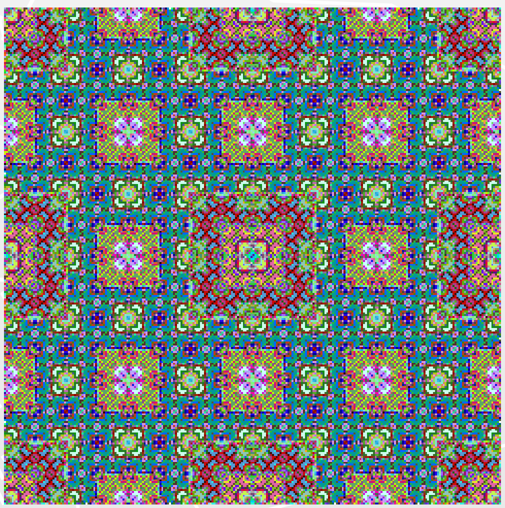


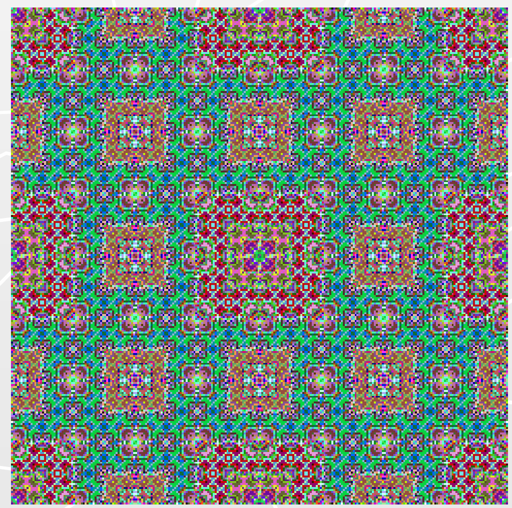
Thousands of unique carpet swatches emerged.

Yes, ***thousands***.

"I kid you not."
--Jack Paar

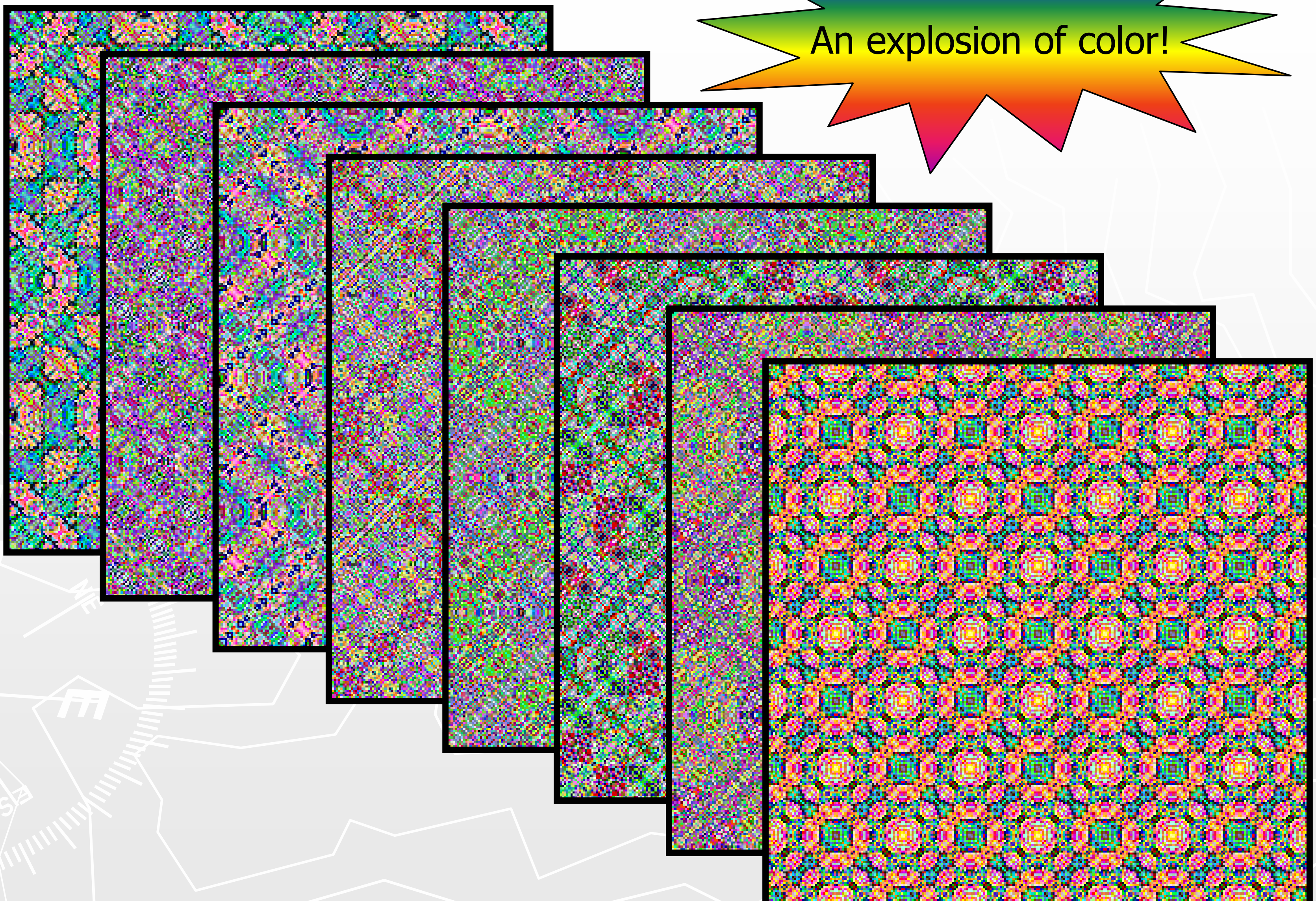
(I wore out the toner cartridge on one of the lab's CalComp printers in no time flat.)





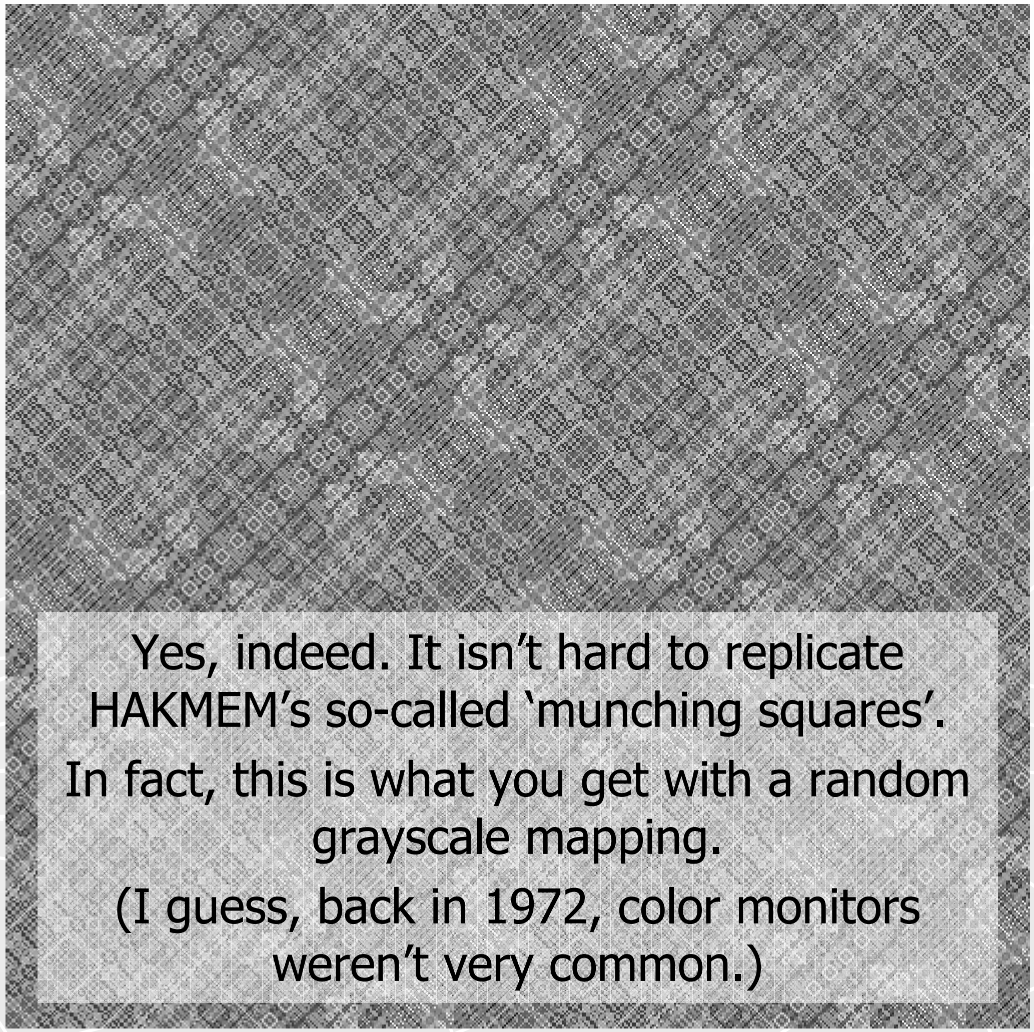
Whoa!

An explosion of color!



Can I replicate HAKMEM?

(Science first…art comes later.)

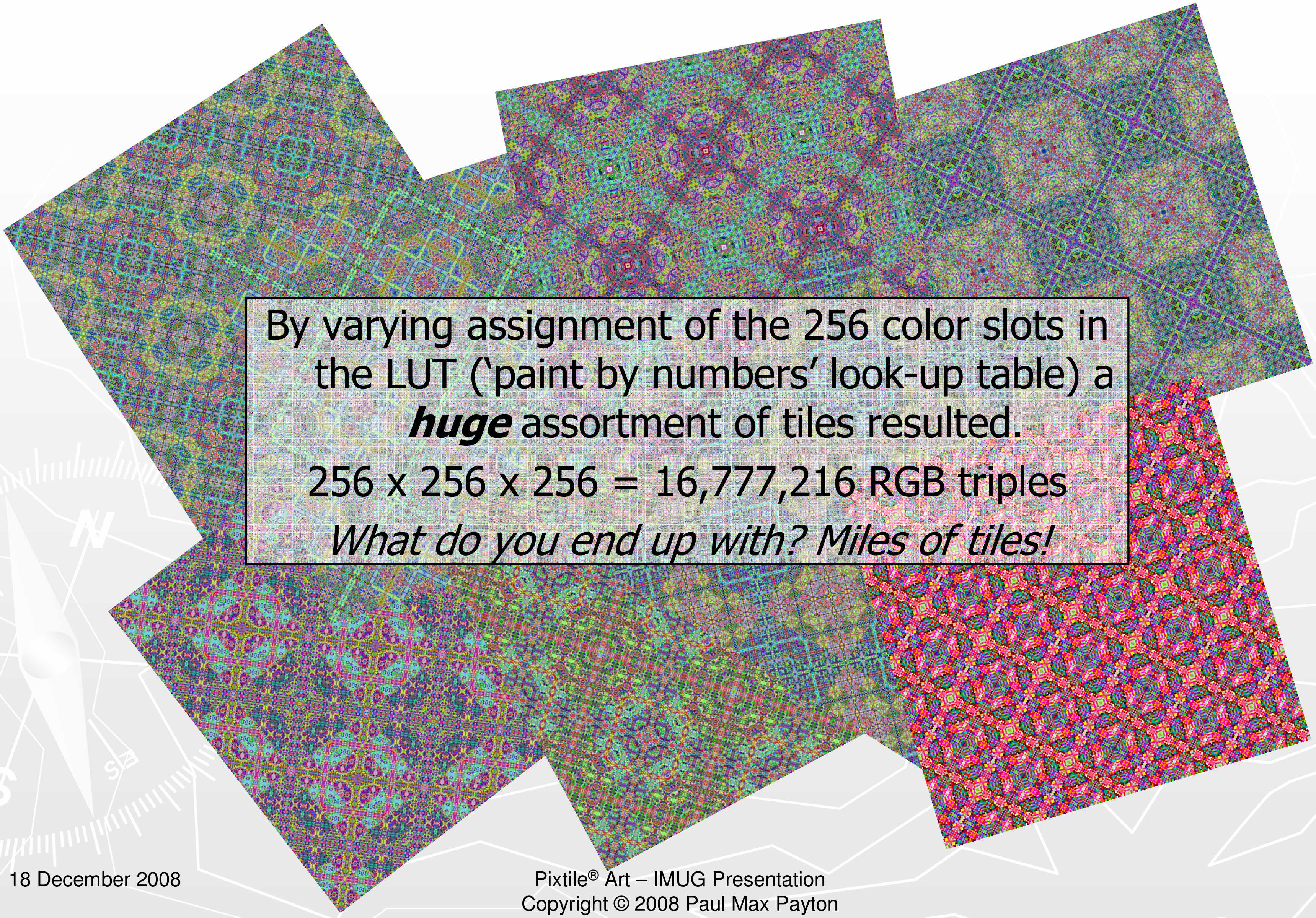


Yes, indeed. It isn't hard to replicate HAKMEM's so-called 'munching squares'.

In fact, this is what you get with a random grayscale mapping.

(I guess, back in 1972, color monitors weren't very common.)

A cornucopia of fabric swatches

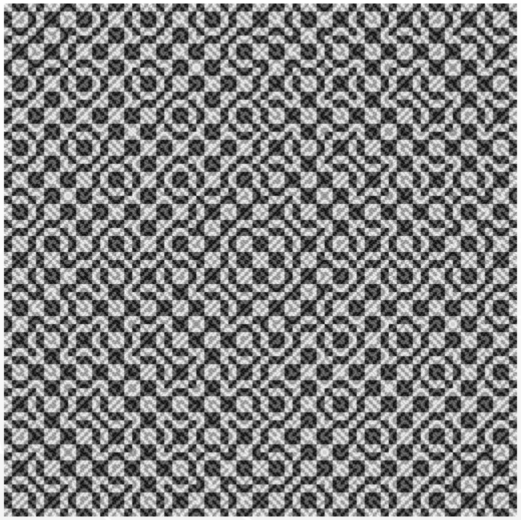


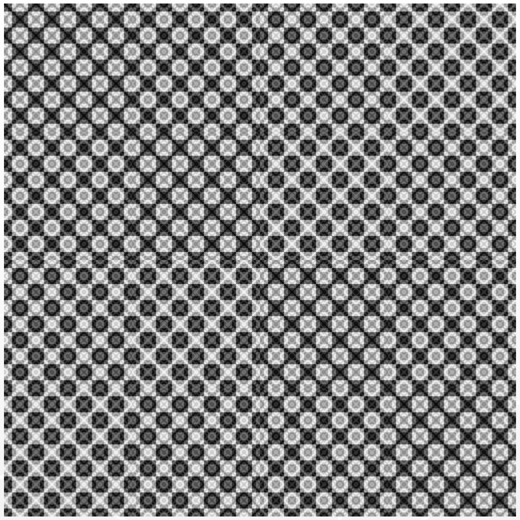
By varying assignment of the 256 color slots in the LUT ('paint by numbers' look-up table) a ***huge*** assortment of tiles resulted.

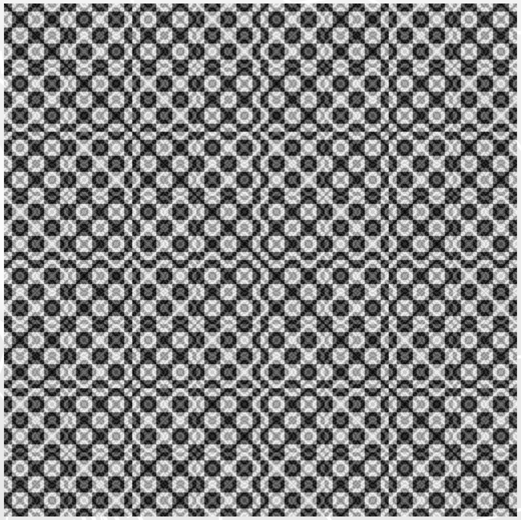
$256 \times 256 \times 256 = 16{,}777{,}216$ RGB triples

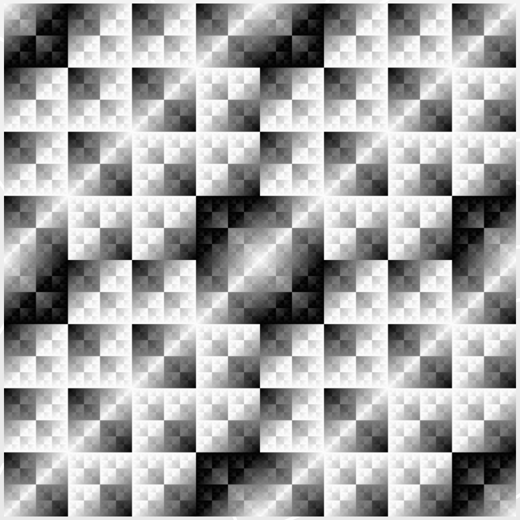
What do you end up with? Miles of tiles!

Prototiles (grayscale)





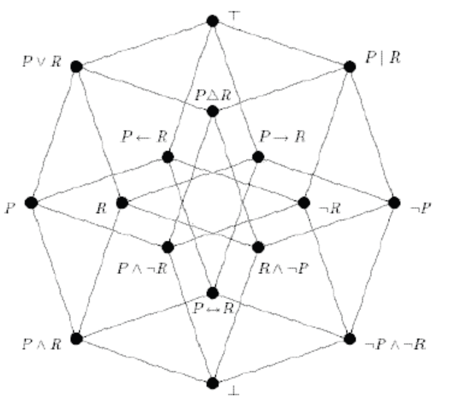




That’s what I call them.

Starting with one of these prototiles, one can assign millions of potential RGB triples to each color slot.

How do I make them? I scramble and mix the bits of my row and column indices. And I then apply any of the 16 possible Boolean operators to those bits.



The Lindenbaum–Tarski algebra of a classical propositional language generated by the two variables $\{P, R\}$.

(You can see where group theory sneaks into things.)

Recursion from iteration?

Why are these tiles so interesting?

One reason is that they have fractal properties (they are self-similar).

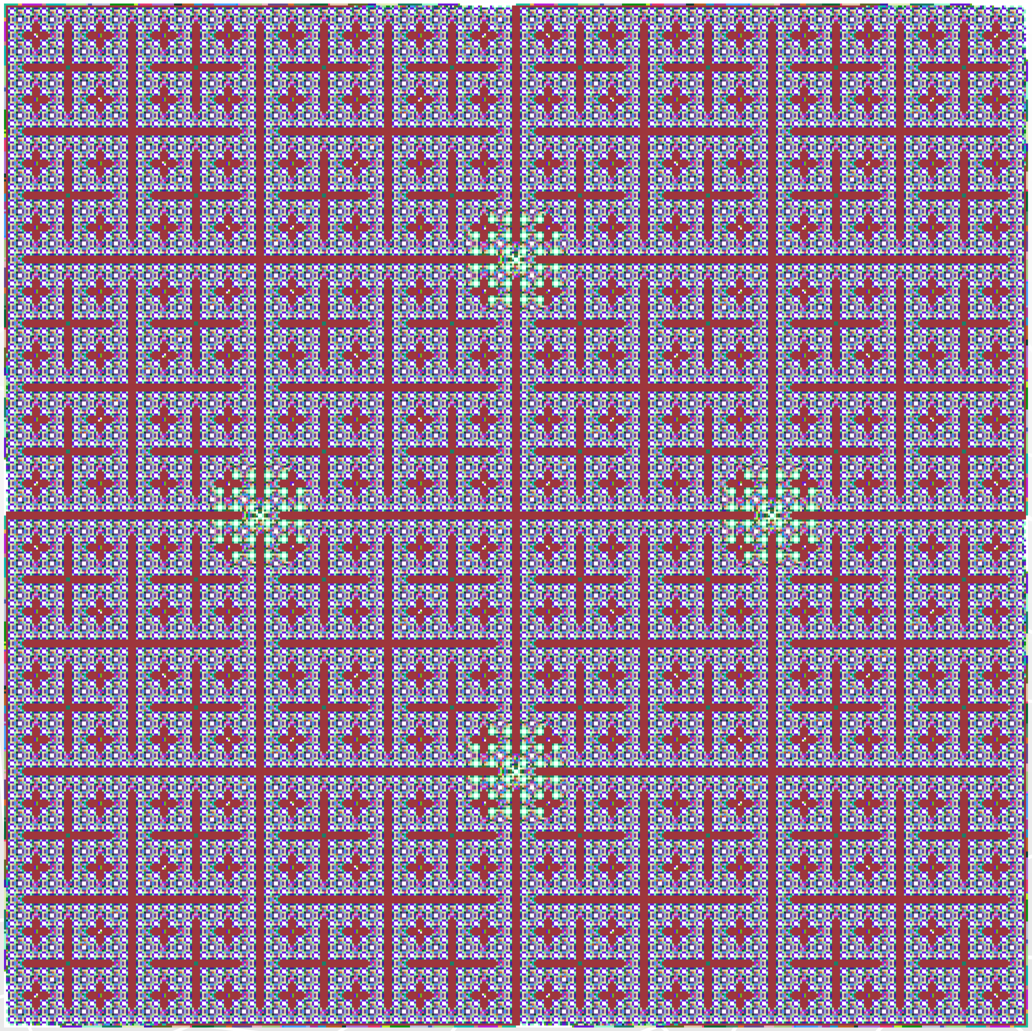
The twist is – they have recursive structure but are produced using iterative means.

Look at this one. It has an innate quad-tree structure to it.

From a statistical point of view, each one of these is indistinguishable from noise.

For the most part, they can't be compressed (high entropy).

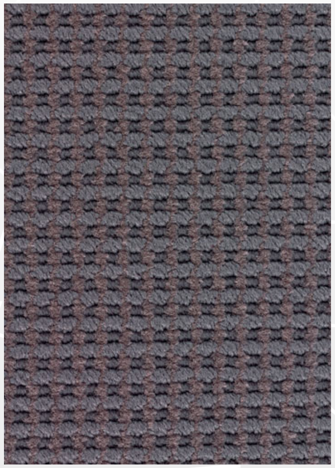
Just try taking the Fourier or wavelet transform of them!



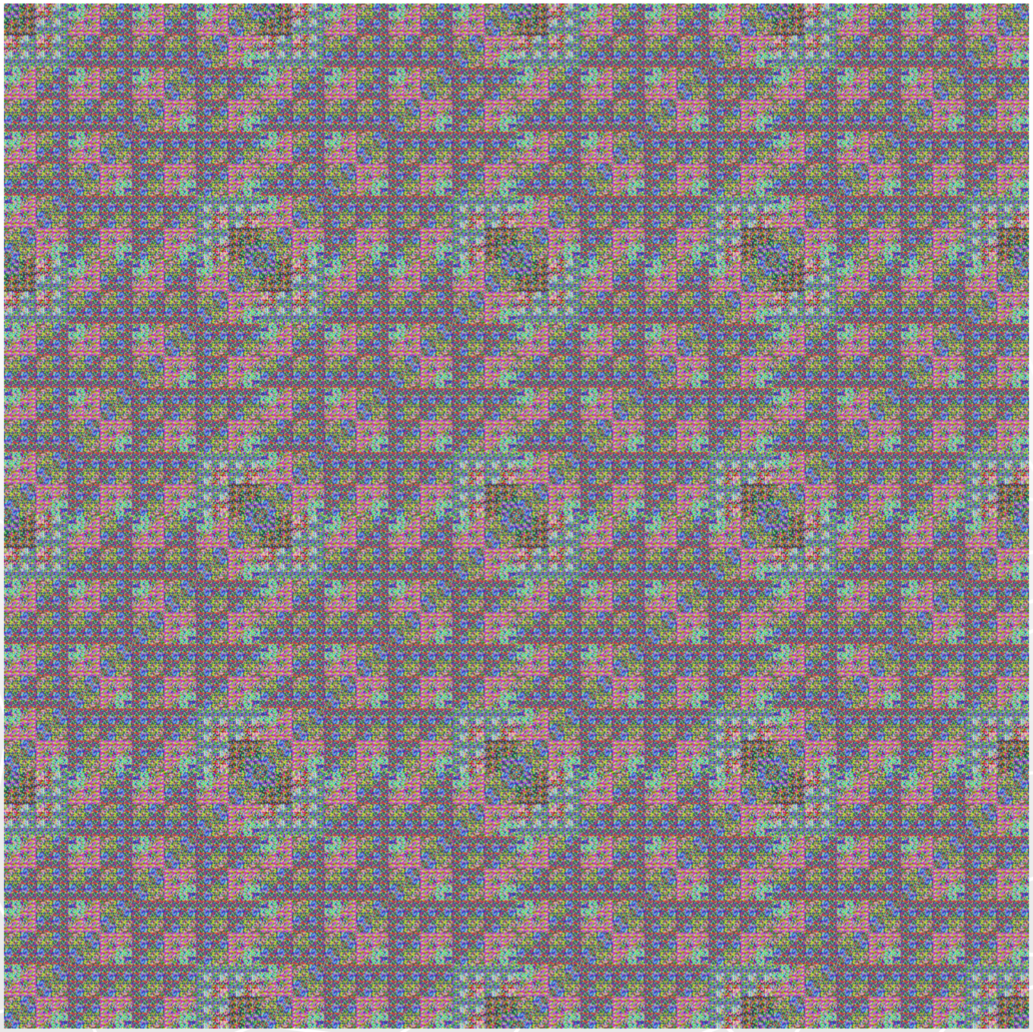
Carpet from a computer? Yes.

Take a look at Mohawk rug tiles and then examine these.

These really ***are*** tiles – they fit together seamlessly in all directions.



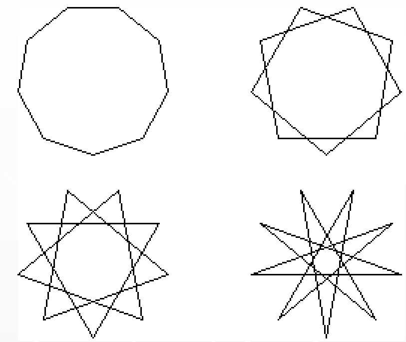
Mohawk® Group
Karastan® Contract
WaffleWeave® 714 Gray Taupe



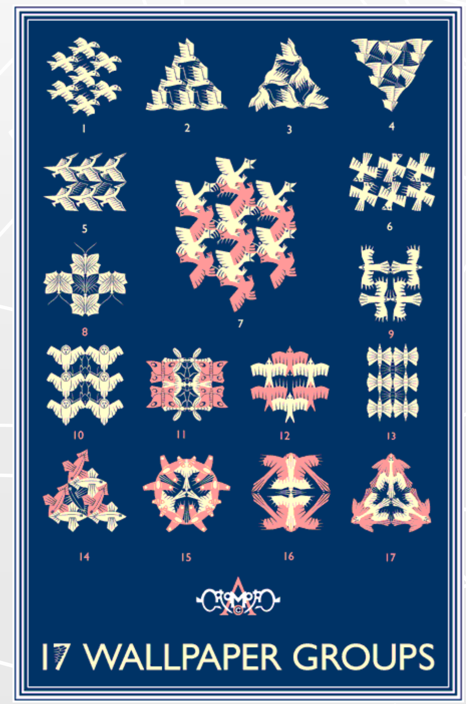
A plethora of questions

All of the preceding is based on squares and rectangles.

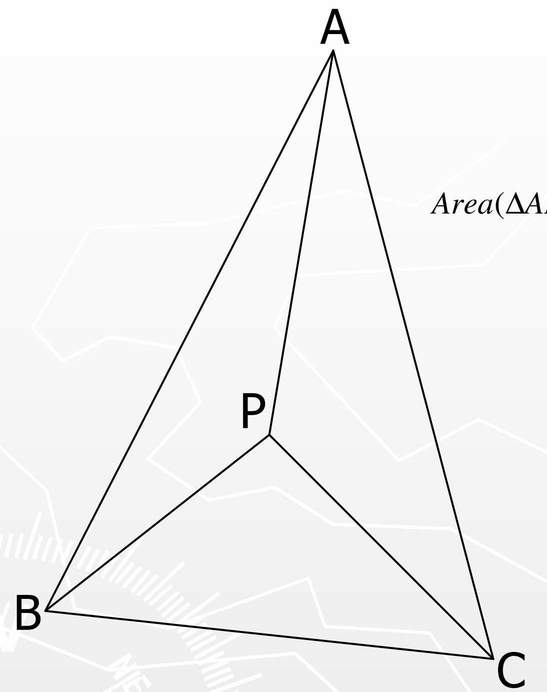
- What about triangles?
  - Rectilinear shapes have row and column indices for pixel coordinates…what coordinates work for triangles?
- What about pentagons, hexagons, … , N-gons?
  - Squares are regular quadrilaterals…what about polygons with greater than four sides? How do coordinates extend for greater generality? What about the non-convex ones?
- What about star polygons?
  - 'Star polygons' is a term used by H.S.M. Coxeter…these are polygons where the 'interior' isn't so clearly defined.
- Do these beauties fit together? [tile/tessellate]
  - We know the squares and rectangles do. Regular triangles and hexagons would as well (honeycombs).
- If so, in how many ways can they fit together?
  - This is the question that consumed Escher most of his life: How many ways are there of tiling the Euclidean plane?
- Is there a way of approaching doing all of this methodically? (the combinatorics here are *killer*…)



{9,1} {9,2} {9,3} {9,4}
star polygons

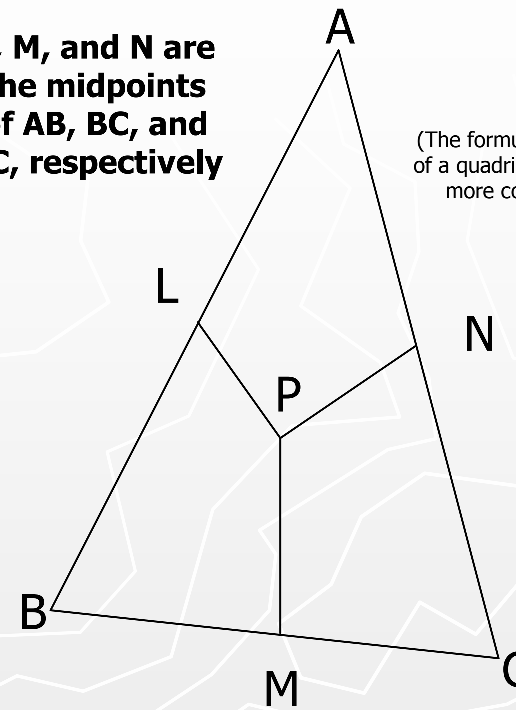


Well, what about triangles?



$$Area(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix}$$

L, M, and N are the midpoints of AB, BC, and AC, respectively



(The formula for the area of a quadrilateral is a little more complicated.)

△ABC = △PBC + △APC + △ABP

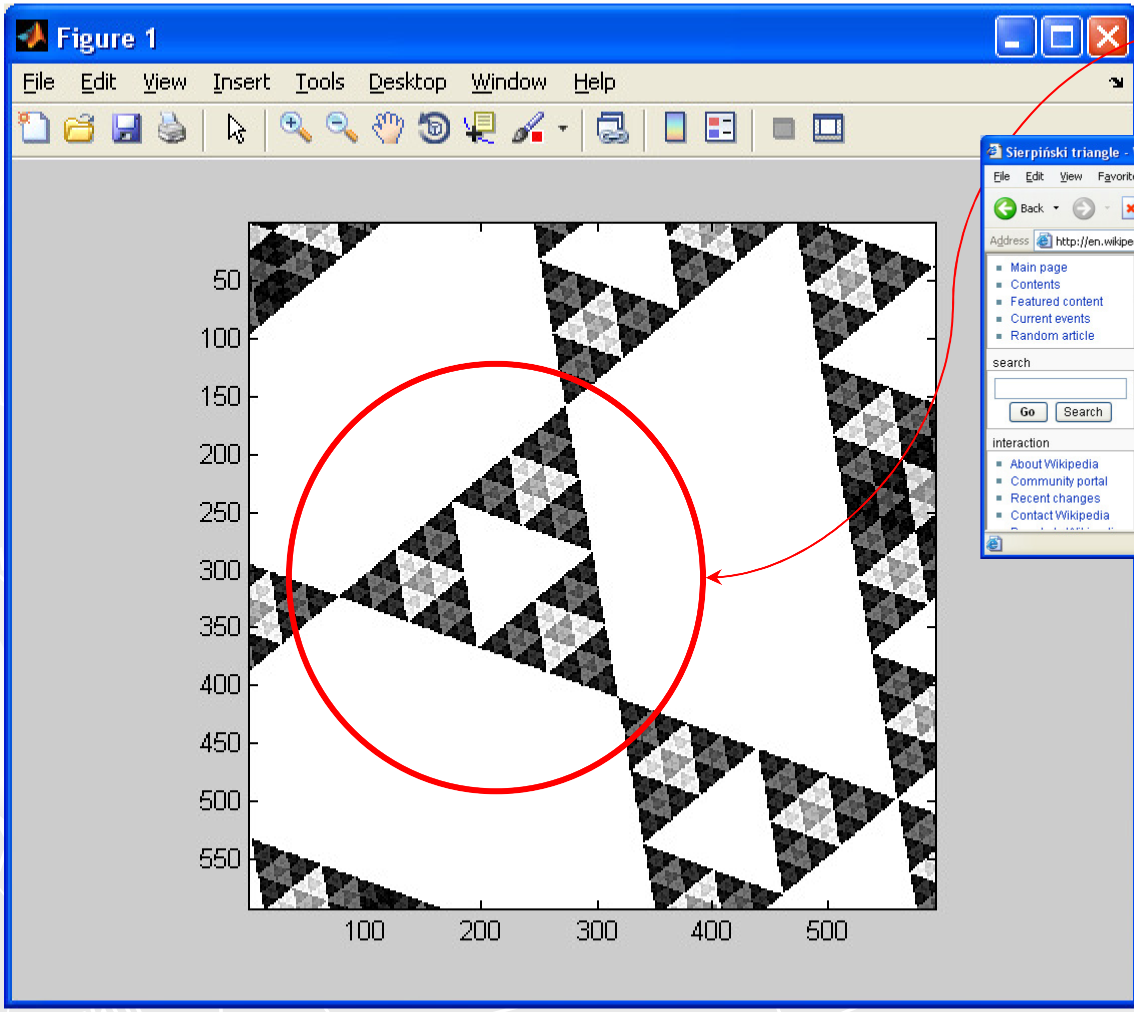
Barycentric/areal homogeneous coordinates
(These are fundamental to finite element analysis.)

△ABC = ☐ALPN + ☐BMPL + ☐CNPM

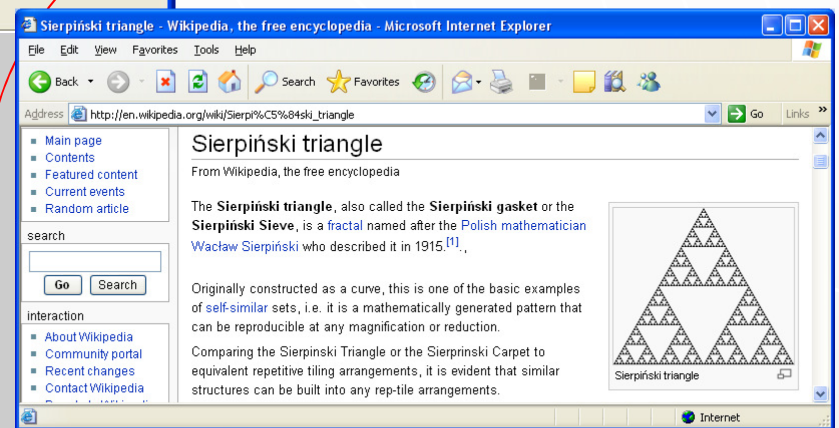
<no name yet> homogeneous coordinates
(No literature exists on these!)

Order of vertices is important for signed areas.

When you XOR areal coordinates…



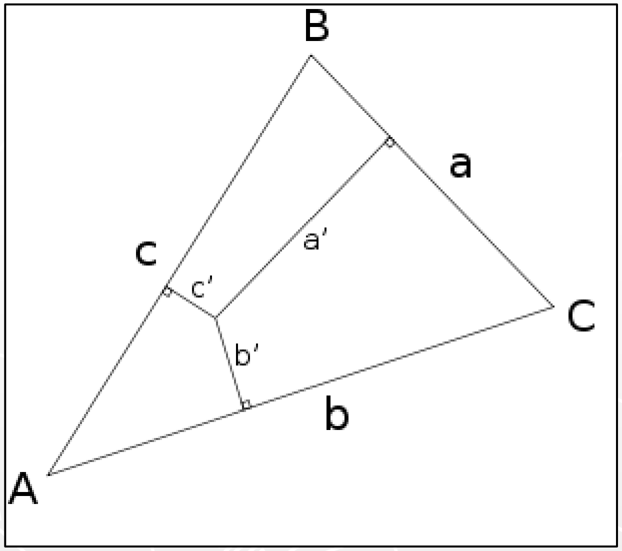
…you get a Sierpinski triangle!

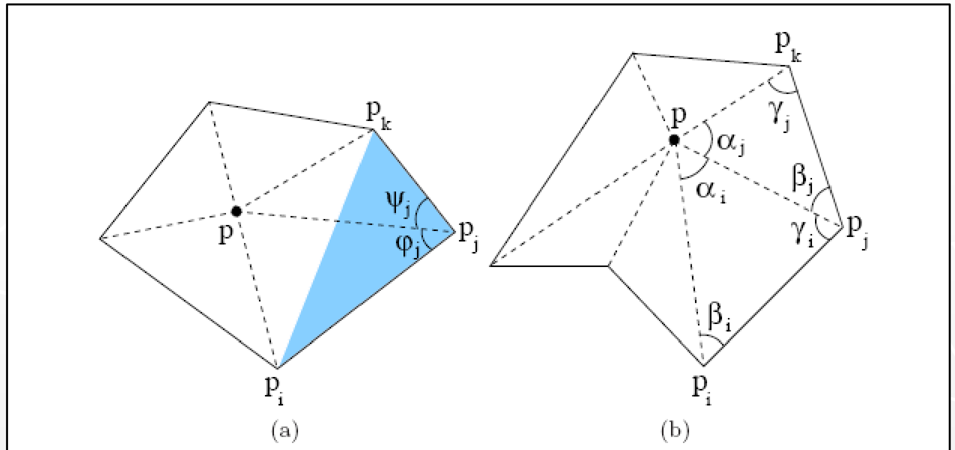


"This is one of the basic examples of *self-similar sets*, i.e. it is a mathematically generated pattern *that can be reproducible at any magnification or reduction*."

"Comparing the Sierpinski Triangle or the Sierpinski Carpet to equivalent repetitive tiling arrangements, it is evident that *similar structures can be built into any arrangement*."

Trilinear coordinates and beyond

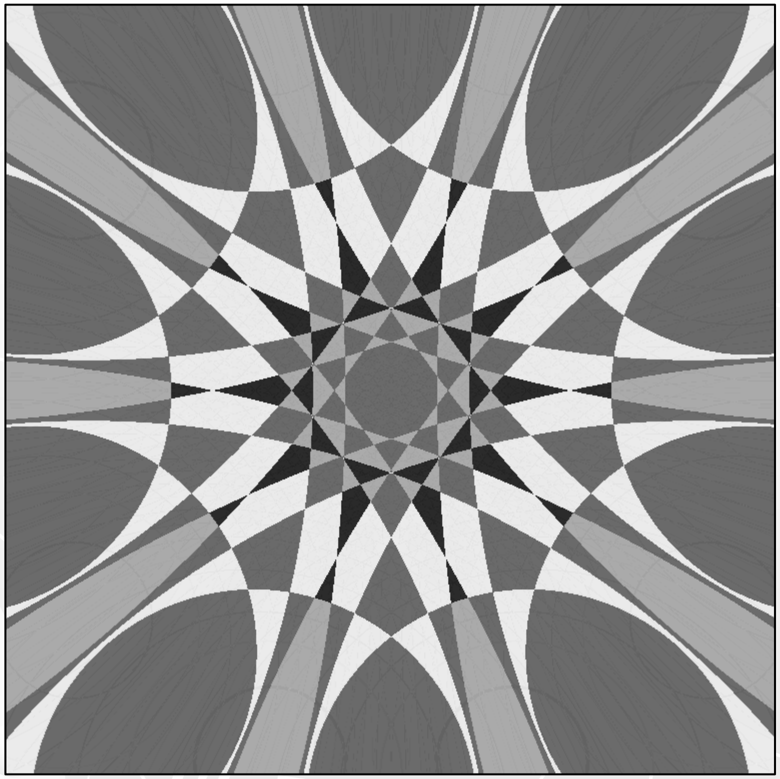




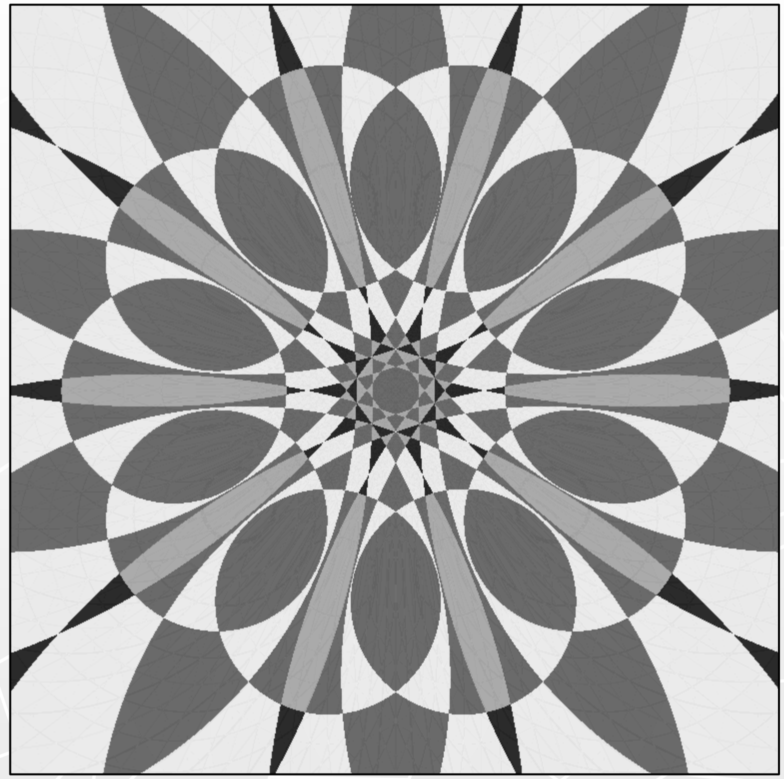
Trilinear coordinates are perpendicular distances to the sides of a reference triangle.

Wachspress and mean value coordinates (Floater) extend the concept of barycentric coordinates to polygons with 3+ sides [and to non-convex ones].

Prototile Beauty from Star Polygons

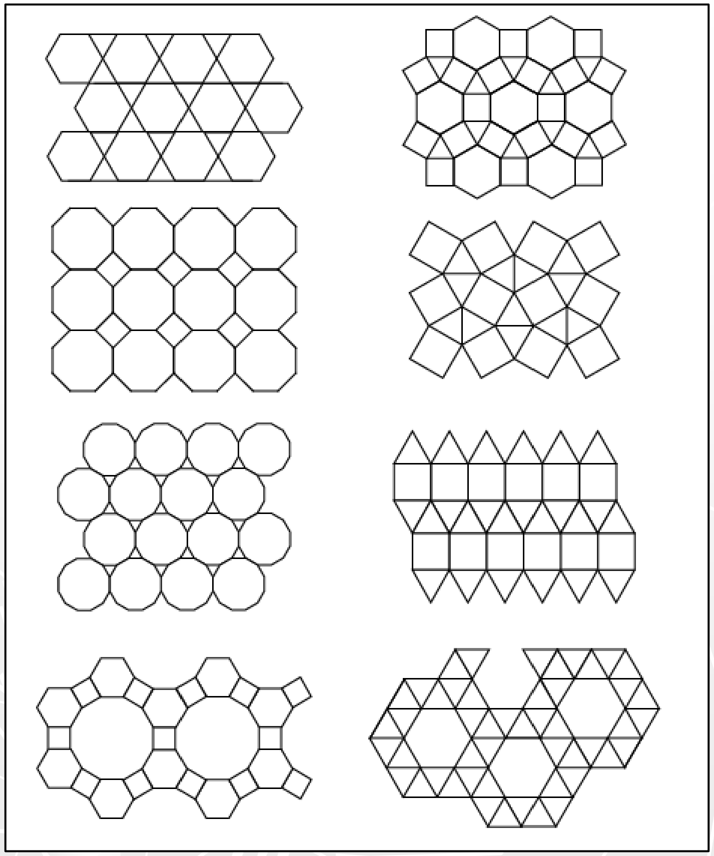


Simple operations on a {10,3} star polygon

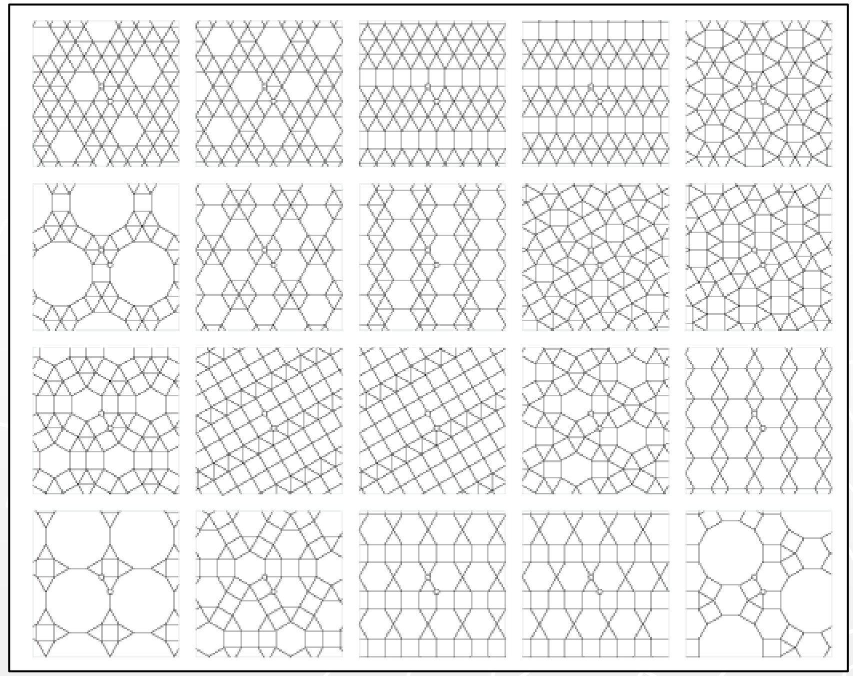


Simple operations inside <u>and</u> outside a {10,2} star polygon

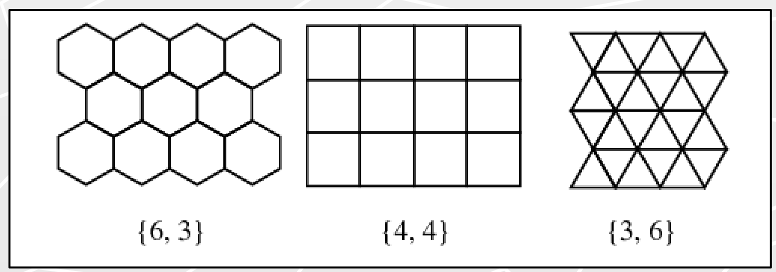
Learning how it all fits together…



Semi-regular tessellations

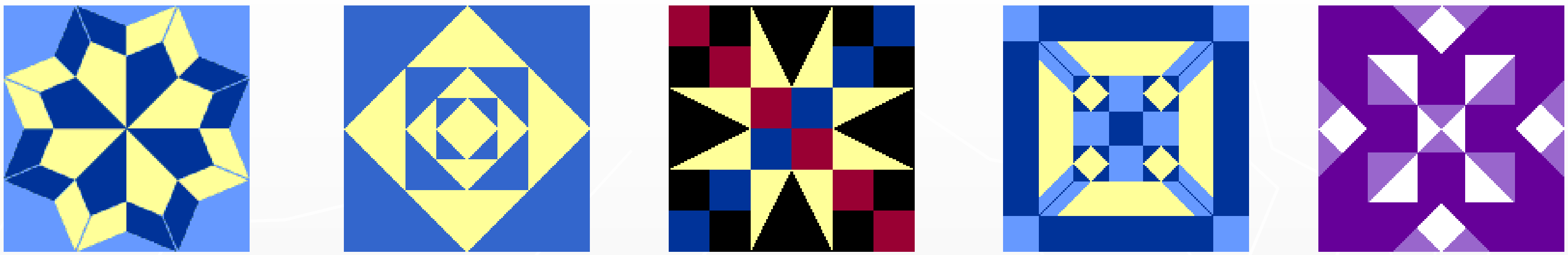


Demi-regular tessellations



Regular tessellations

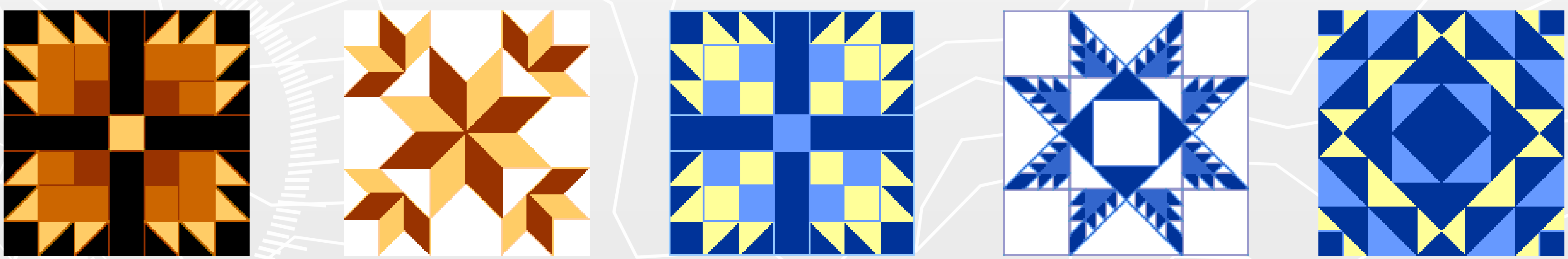
Quilt Blocks: Geometry all sewn up



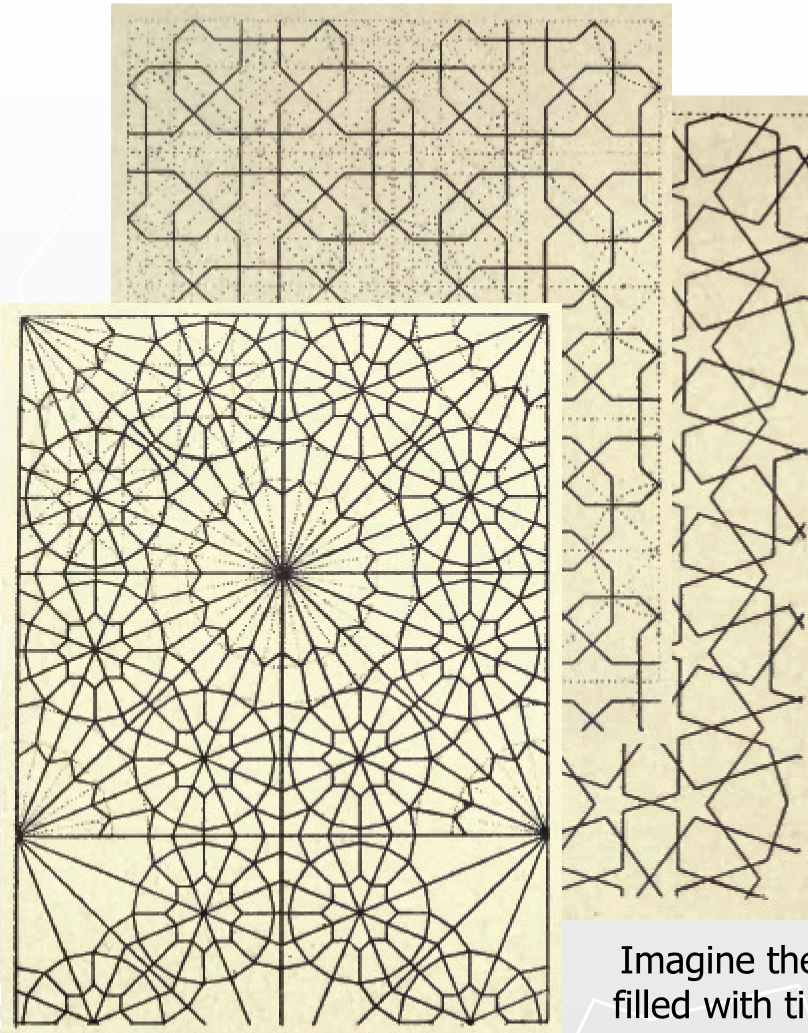
Barbara Brachman's "Encyclopedia of Quilt Patterns" has 6,500+ quilt blocks.

Maggie Malone lists 5,500+ quilt blocks in her "Quilt Block Designs".

Mental Image: Fill each polygon with a tile pattern. Cyber-tapestries!



Islam: A Genius for Geometry



Imagine these filled with tiles!

The Alhambra in Granada, Spain
(14th century Muslim caliphate)

Sura 21:52 in the Qur’an forbids devout Muslims from making images of animals and people. Thanks to this edict of aniconism, Islam has given us some of the most beautiful examples of geometric art (arabesque) ever seen.

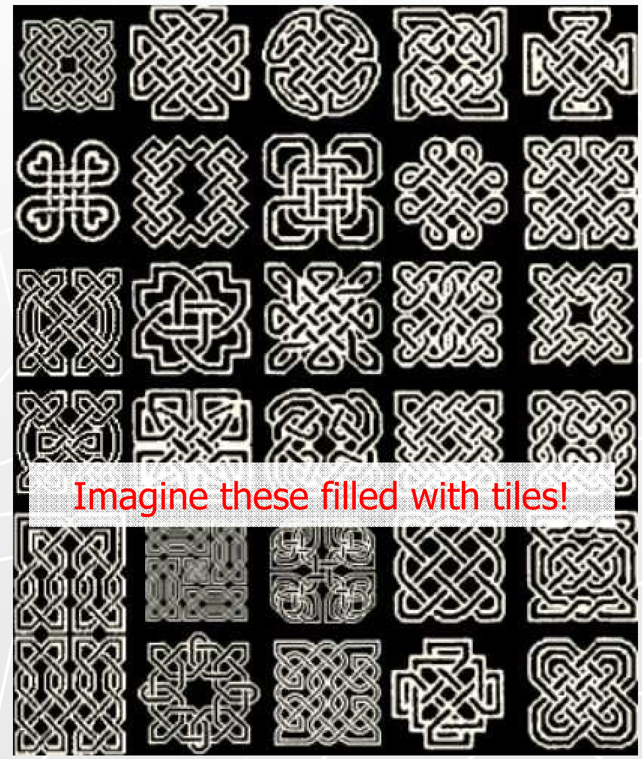
You may take issue with their beliefs or their extremists…but their artistry is undeniable.

Celtic Art: Knots and Plaits

Celtic knots and ‘step patterns’ date back to 450 AD and pre-date Christian influence on the Celts. The style is commonly associated with Celtic lands but it was also practiced extensively in England and exported to Europe by Irish and Northumbrian monks. Celtic Art is popularly thought of in terms of national identity and therefore specifically Irish, Scottish or Welsh.

Lindisfarne Gospels (698-721)

Part of The Great Pavement, a Roman mosaic laid in AD 325 at Woodchester, Gloucestershire, England.



UNICODE for Crystallography

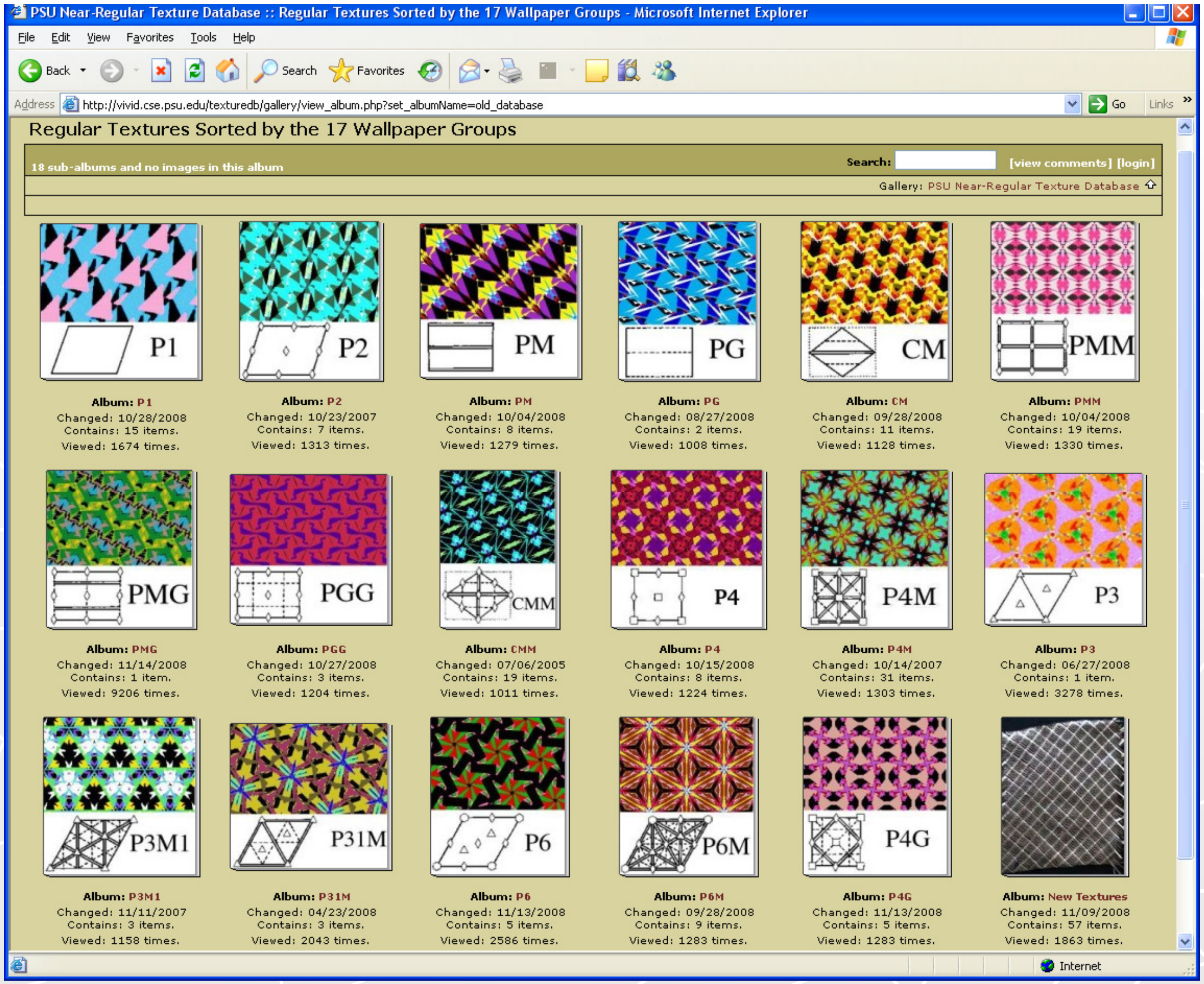
"Standards are a good thing. Let's have lots of them!"

Crystallographers care deeply about the lattice structure of the world and use notation to categorize it.

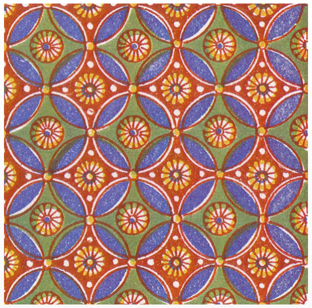
Interestingly, the 230 3D space groups were well-understood before the 17 2D wallpaper groups were!

Here, a catalog of the Hermann-Mauguin symbols for the 2D wallpaper groups is shown with some examples and unit cells.

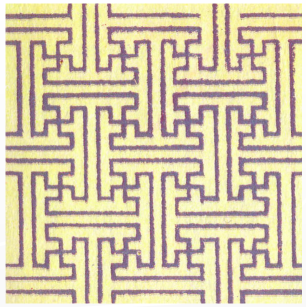
The notation represents mirror reflections (m), glide reflections (g), and a 'p' or a 'c' for primitive unit cell.



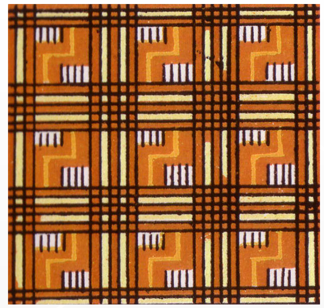
The World Speaks Geometry



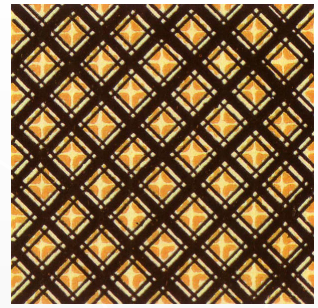
p4m (Egypt)



p4g (China)



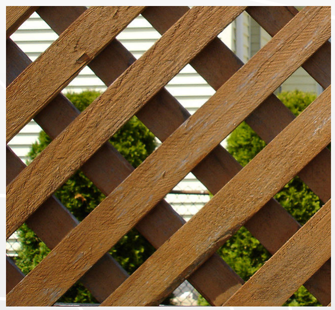
p2 (Hawaii)



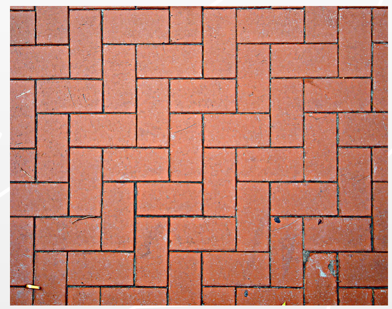
p4m (Tahiti)

pm (India)

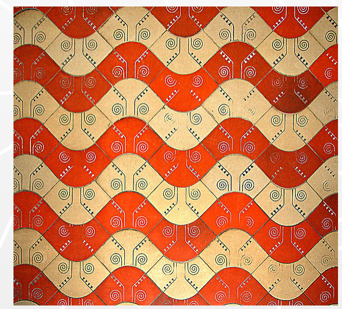
cm (Assyria)



pmm (fence)

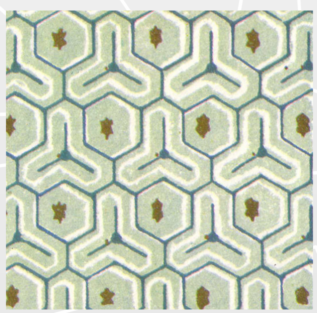


pgg (Hungary)

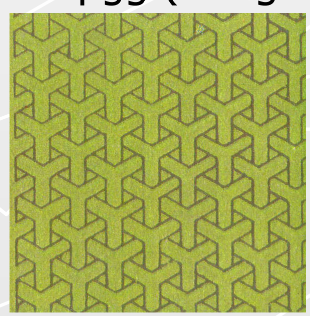


pmg (Czech Republic)

cmm (Turkey)



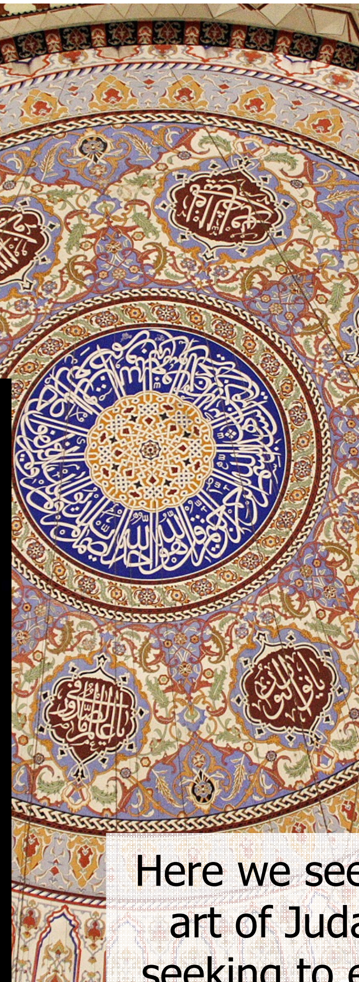
p3m (Persia)



p31m (China)

p3m1 (Hungary)

Art and Science Unify Humanity



Here we see the art of geometry inform religious art of Judaism, Islam, and Christianity. When seeking to exalt the ineffable, humanity looks to the universal beauty of geometry for expression.

"God is a geometer." – Pythagoras, Plato, Kepler

What next? A mind-bending ‘to-do list’

- Of immediate interest/utility
  - Discrete harmonic coordinates (Floater/Hormann/Kos)
  - Complementary algebra (Frederick M. Lewis)
  - Convolutions with kernels and neighborhood operators
  - Various color systems (e.g., CMYK, YIQ, YUV, CIE systems)
  - Fuzzy, polyvalent (Sugeno/Yager), and continuous-value logic
  - Regular, semi-regular, demi-regular, quasi-regular tessellation
  - Constructive geometric operations
  - Star polygons (H.S.M. Coxeter)
  - Geometric transformations (conformal, anamorphic, non-orthogonal, isometries)
  - Overlays and inlays (layering)
  - FEM iso-parametric node-based indexing
  - Different distance metrics
  - Encyclopedia of Triangle Centers (Clark Kimberling)
  - Mixing and matching coordinates
- Of later interest/utility
  - Walsh/Haar/Hadamard patterns
  - Digital watermarks and embedded codings
  - Celtic knots
  - Lindenmayer systems
  - Voronoi and Delaunay tessellations (and duals)
  - FFT and other image filters
  - Genetic algorithms and Markov random fields
  - Automata theory
  - Cellular automata (Wolfram, Toffoli)
  - Quilt/needlepoint/cross-stitch grammars
  - Truchet tilings
  - Hexagonal and triangular lattices
  - Flip-flops and time-delay logic
  - Penrose tilings
  - Handbooks of ornament (Owen Jones, Franz Sales Meyer)
  - Culbertson’s “nerve nets”
  - Parabolic trigonometry
  - Non-Euclidean geometry (Klein-Beltrami, Poincare)

Please observe something subtle, yet purposefully symbolic.

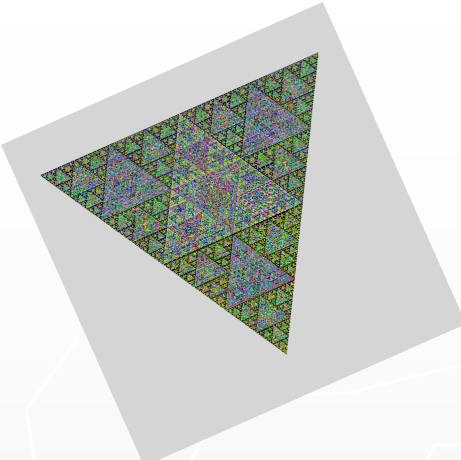
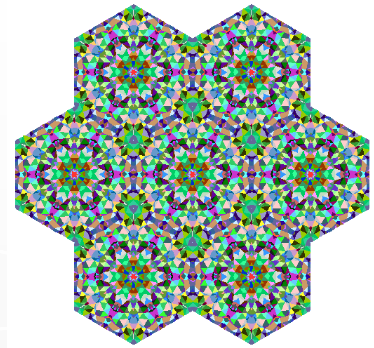
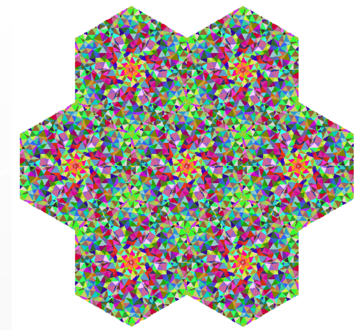
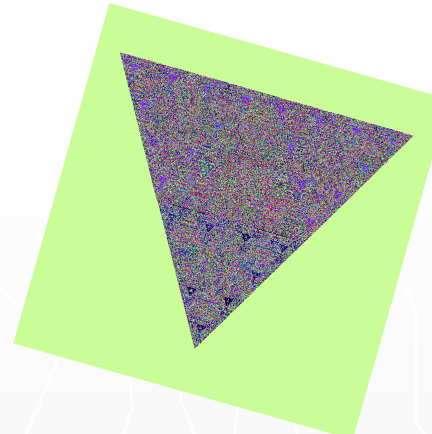
I used a compass motif as a background because, in a very real sense, much of this work is exploring unknown territory or combining established ideas in ways that were never imagined before!

Thank you!

"Mathematics, rightly understood, possesses not only truth, but supreme beauty." – Bertrand Russell

"It was an endless palace full of pools and waterfalls, tumbling into dull and burnished gold." -- Charles Baudelaire

"These are not spring flowers, at the mercy of the changing seasons, but rather never-fading amaranths, gathered from the most beautiful flowerbeds of geometry." -- Blaise Pascal

“And awaaay we go…”

(now, a tour de force of the algorithmic art)